

Effective field theory approach for the M1 properties of $A=2$ and 3 nuclei

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Abstract

The magnetic moments of ^2H , ^3He and ^3H as well as the thermal neutron capture rate on the proton are calculated using heavy baryon chiral perturbation theory *à la* Weinberg. The M1 operators have been derived up to $N^3\text{LO}$. The nuclear matrix elements are evaluated with the use of wave functions obtained by carrying out variational Monte Carlo calculations for a realistic nuclear Hamiltonian involving high-precision phenomenological potentials like Argonne Av18 and Urbana IX tri-nucleon interactions. We discuss the potential- and cutoff-dependence of the results.

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1 Introduction

The standard nuclear physics approach (SNPA) based on meson-exchange currents[1], high-precision phenomenological potentials and the state-of-the-art techniques for obtaining a few-body nuclear wave functions has achieved tremendous progress in understanding light nuclear systems[2]. SNPA is particularly useful when the impulse contributions and the well-known long-ranged one-pion-exchanges predominate, as was formulated in terms of the so-called *chiral filtering* [3]. However, if more complicated meson exchange processes are important, SNPA becomes model-dependent because the phenomenological description of the nuclear potentials and transition operators does not allow the unique description of their short-range behavior.

The merit of effective field theory (EFT) [4] is that it offers a systematic way to control short-range physics, which in principle allows more accurate theoretical predictions than achievable in the conventional approach [5]. An example is the *hep* process, ${}^3\text{He} + p \rightarrow {}^4\text{He} + \nu_e + e^+$, which is potentially important for solar neutrino physics. A reliable estimation of the *S*-factor for this reaction has been a long-standing challenge in nuclear physics [6]; its theoretical estimates have varied by orders of magnitude in the literature. A highly elaborate SNPA calculation of this *S*-factor was carried out by Marcucci *et. al.* [7], but it was a recent EFT-based calculation by Park *et al.* [8] that provided definitive support to the SNPA results, giving in addition a quantitative error estimate. In [8], the authors have developed an EFT approach, called EFT* or MEEFT (*more effective* effective field theory) and performed a parameter-free calculation of the *hep* cross section, which has led to an estimate of the *S*-factor with $\sim 15\%$ precision; for a detailed review, see Ref. [9]. The same method has marked successes also in describing the highly suppressed isoscalar amplitude pertinent to the $n+p \rightarrow d+\gamma$ process [10, 11], as well as the weak processes like muon capture on the deuteron [12] and ν -*d* scattering [13, 14]. For a recent review, see Ref. [15].

The strategy of MEEFT, as explained in [9, 8, 16], is in the spirit of the original Weinberg scheme [17] based on the chiral expansion of *irreducible* terms. In MEEFT the relevant current operators are derived systematically by applying HB χ PT to a specified order. Nuclear matrix elements corresponding to these current operators are evaluated with the use of realistic nuclear wave functions obtained by applying an ab-initio or quasi-ab-initio few-body calculation method to a realistic phenomenological nuclear Hamiltonian that involves high-precision phenomenological potentials. In MEEFT, as in the usual EFT, short-range physics is simulated by contact counter-terms and the coefficients of these terms, called the low-energy constants (LECs), are determined by requiring that a selected set of experimental data is reproduced. This renormalization procedure is expected to remove, to a large extent, model-dependence that might creep in through the phenomenological parametrization of short-range physics in the adopted nuclear interaction. We note that the strategy employed here is closely related to the one used to construct a universal V_{low-k} in the recently developed renormalization-group approach to nuclear interactions [18].

The purpose of this letter is to demonstrate that MEEFT provides a practical and reliable tool to compute the low energy M1 properties of few-nucleon systems

in a largely model-independent way. We calculate here four low-energy electromagnetic observables, namely the magnetic moments of ${}^2\text{H}$, ${}^3\text{He}$ and ${}^3\text{H}$ and the rate of radiative capture of a thermal neutron on a proton. We derive all the relevant operators up to the next-to-next-to-next leading order (N^3LO) of the perturbation series, which include the short-range contact terms. We use the experimental values of two of the above-mentioned four observables to fix the coefficients of the contact terms appearing at N^3LO and make prediction for the remaining two observables. We test the consistency as well as the model-independence of the obtained results. To our knowledge, this is the first EFT calculation of the magnetic moments of the $A=3$ systems.

Before closing this section, we remark that other EFT methods can also be adopted, at least in principle, for studying the M1 properties of few-nucleon systems at low energy. One of such possible alternatives consists in deriving not only the transition operators but also the nuclear interactions using the same EFT. This method has the advantage of being more transparent in discussing the order counting and model-independence; for the recent developments in the EFT studies of nuclear potentials, see Refs. [19, 20, 21] and references therein. Such a study is expected to be highly illuminating particularly for the electric transitions because gauge invariance (or charge conservation) is automatically guaranteed order by order. Another possibility is to use the so-called pionless EFT, where even the pions are integrated out leaving only the nucleon field as a pertinent degree of freedom. Pionless EFT has been successfully applied to some processes involving two- or three-nucleon systems [22].

2 Current operators

The relevant operator for the magnetic moment and radiative np capture at threshold is the M1 operator $\boldsymbol{\mu}(q)$,

$$\boldsymbol{\mu}(q) \equiv \left(\frac{i\mathbf{q}}{\sqrt{6\pi}} \right)^{-1} \hat{T}_{10}^{Mag}(q) \quad (1)$$

where $\hat{T}_{10}^{Mag}(q)$ is defined in [23], $q^\mu = (\omega, \mathbf{q})$ is the momentum carried out by the photon ($q^\mu = 0$ for the magnetic moment while $q^\mu \neq 0$ for np capture), and $q \equiv |\mathbf{q}|$.

We derive the M1 operator in $\text{HB}\chi\text{PT}$, which contains the nucleons and pions as pertinent degrees of freedom with all other massive fields integrated out. In $\text{HB}\chi\text{PT}$ the electromagnetic currents and M1 operator are expanded systematically with increasing powers of Q/Λ_χ , where Q stands for the typical momentum scale of the process and/or the pion mass, and $\Lambda_\chi \sim 4\pi f_\pi \sim m \sim 1 \text{ GeV}$ is the chiral scale, $f_\pi \simeq 92.4 \text{ MeV}$ is the pion decay constant, and m is the nucleon mass. We remark that, while the nucleon momentum \mathbf{p}_i is of order of Q , its energy ($\sim \mathbf{p}_i^2/m$) is of order of Q^2/m , and consequently the four-momentum of the emitted photon q^μ (with $\omega = |\mathbf{q}|$) should also be counted as $\mathcal{O}(Q^2/m)$. In this work we include *all* the contributions up to N^3LO , where N^νLO denotes terms of order of $(Q/\Lambda_\chi)^\nu$ compared to the leading one-body contribution. It is worth mentioning that there is a different

power counting scheme where the nucleon mass is regarded as heavier than the chiral scale, see Refs. [19, 20] for details. However, the use of this alternative counting scheme would not affect the results to be reported in this article since the difference between the two counting schemes would appear only at orders higher than explicitly considered here (N³LO).

The one-body (1B) M1 operator including the relativistic corrections reads

$$\begin{aligned}\boldsymbol{\mu}_{1B}(q) = & \sum_i \frac{1}{2m_p} \left\{ \hat{j}_0(qr_i) \left[\boldsymbol{\sigma}_i \left(\mu_i - Q_i \frac{\bar{\mathbf{p}}_i^2}{2m^2} \right) - \frac{\mu_i - Q_i}{2m^2} \bar{\mathbf{p}}_i \boldsymbol{\sigma}_i \cdot \bar{\mathbf{p}}_i \right] \right. \\ & + \hat{j}_1(qr_i) \left[Q_i \mathbf{r}_i \times \bar{\mathbf{p}}_i \left(1 - \frac{\bar{\mathbf{p}}_i^2}{2m^2} \right) - \frac{w(2\mu_i - Q_i)}{4m} i \mathbf{r}_i \times (\bar{\mathbf{p}}_i \times \boldsymbol{\sigma}_i) \right] \\ & \left. + \frac{(qr_i)^2}{30} \hat{j}_2(qr_i) (3\hat{\mathbf{r}}_i \hat{\mathbf{r}}_i \cdot \boldsymbol{\sigma}_i - \boldsymbol{\sigma}_i) + \dots \right\}\end{aligned}\quad (2)$$

where $\hat{j}_n(x) \equiv \frac{(2n+1)!!}{x^n} j_n(x) = 1 + \mathcal{O}(x^2)$, $Q_i = (1 + \tau_i^z)/2$ is the charge of the i -th nucleon, and $\mu_i = (\mu_s + \tau_i^z \mu_v)/2$ is the magnetic moment in units of the nuclear magneton, $\mu_N = e/(2m_p)$, with $\mu_s = \mu_p + \mu_n \simeq 0.8798$ and $\mu_v = \mu_p - \mu_n \simeq 4.7059$; $\bar{\mathbf{p}}_i \equiv \frac{1}{2}(i \overleftrightarrow{\nabla}_i - i \overleftrightarrow{\nabla}_i)$ with the understanding that the derivatives act only on the wave functions, and $r_i \equiv |\mathbf{r}_i|$. In the above equation the familiar $(\mu_i \boldsymbol{\sigma}_i + Q_i \mathbf{r}_i \times \bar{\mathbf{p}}_i)$ term is of leading order (LO), while all the others terms are N²LO. Since there are no N³LO contributions to the $\boldsymbol{\mu}_{1B}$, the neglected terms are of N⁴LO and higher orders.

Corrections to the 1B operator are due to the meson-exchange currents (MEC). Up to N³LO, only two-body (2B) contributions enter; three-body (3B) currents are N⁴LO or higher order. It is to be emphasized that MECs derived in EFT are meaningful only up to a certain momentum scale Λ , where Λ is the cutoff below which the chosen explicit degrees of freedom reside. This cutoff may be realized by introducing a Gaussian regulator in performing the Fourier transformation of the MECs from momentum space to coordinate space [8]. It is to be noted that the contributions due to high momentum exchanges (above the cutoff scale) are not simply ignored but, as we will discuss later, they are accounted for by the renormalization of the contact-term coefficients.

We decompose the two-body current into the *soft*-one-pion-exchange (1π) term, vertex corrections to the one-pion exchange ($1\pi C$) term, the so-called *fixed* term contribution ($1\pi : \text{fixed}$), the two-pion-exchanges (2π) term, and the contact-term contribution (CT),

$$\boldsymbol{\mu}_{2B}(q) = \sum_{i < j} \left[\boldsymbol{\mu}_{ij}^{1\pi} + \left(\boldsymbol{\mu}_{ij}^{1\pi C} + \boldsymbol{\mu}_{ij}^{1\pi: \text{fixed}} + \boldsymbol{\mu}_{ij}^{2\pi} + \boldsymbol{\mu}_{ij}^{CT} \right) \right] = \text{NLO} + \text{N}^3\text{LO}. \quad (3)$$

The leading MEC due to the *soft*-one-pion-exchange (1π) is NLO and given as

$$\begin{aligned}\boldsymbol{\mu}_{12}^{1\pi} = & \frac{g_A^2}{8f_\pi^2} \left[\hat{T}_S^{(\times)} \left(\frac{2}{3} y_{1\Lambda}^\pi(r) - y_{0\Lambda}^\pi(r) \right) - \hat{T}_T^{(\times)} y_{1\Lambda}^\pi(r) \right] \hat{j}_0(qR) \\ & - \frac{g_A^2 m_\pi^2}{24f_\pi^2} \tau_\times^z \mathbf{R} \times \mathbf{r} [\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \bar{y}_{0\Lambda}^\pi(r) + S_{12} y_{2\Lambda}^\pi(r)] \hat{j}_1(qR) + \dots, \quad (4)\end{aligned}$$

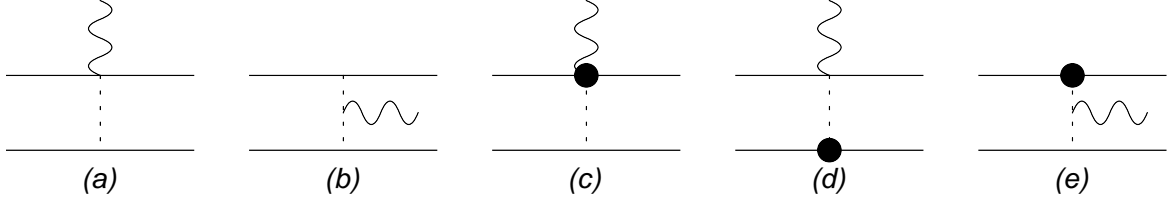


Figure 1: Tree graphs of NLO and N³LO. One-pion exchange “seagull” (a) and “pion-pole” diagram (b) contribute to the $\mu^{1\pi}$. Diagrams (c)-(e) contribute to the $\mu^{1\pi C}$ and $\mu^{1\pi;fixed}$ at N³LO. The dot represents the vertex corrections coming from NLO or N²LO lagrangian.

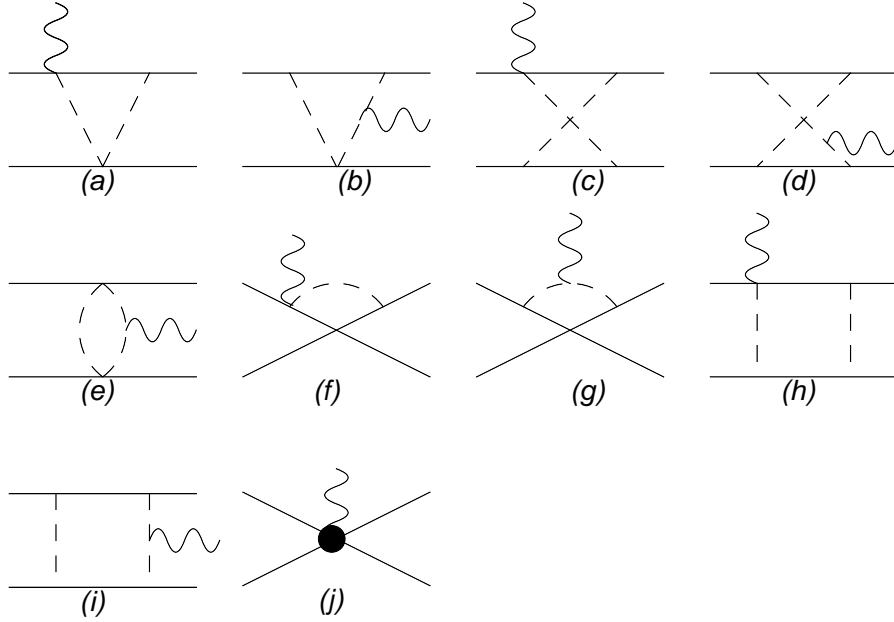


Figure 2: Diagrams contributing to $\mu^{2\pi}$ (a)-(i) and μ^{CT} (j) at N³LO.

where $g_A \simeq 1.2695$, $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$, $r = |\mathbf{r}|$, $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2$, $R = |\mathbf{R}|$, and

$$\begin{aligned}\delta_\Lambda(r) &= \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{-\mathbf{k}^2/\Lambda^2} e^{i\mathbf{k}\cdot\mathbf{r}}, \\ y_{0\Lambda}^\pi(r) &= \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{-\mathbf{k}^2/\Lambda^2} e^{i\mathbf{k}\cdot\mathbf{r}} \frac{1}{\mathbf{k}^2 + m_\pi^2},\end{aligned}\quad (5)$$

$y_{1\Lambda}^\pi = -r \frac{d}{dr} y_{0\Lambda}^\pi$, $y_{2\Lambda}^\pi = \frac{r}{m_\pi^2} \frac{d}{dr} \frac{1}{r} \frac{d}{dr} y_{0\Lambda}^\pi$ and $S_{12} = 3\boldsymbol{\sigma}_1 \cdot \hat{\mathbf{r}} \boldsymbol{\sigma}_2 \cdot \hat{\mathbf{r}} - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$. We have also defined $\hat{T}_S^{(\odot)} \equiv \tau_\odot^z \boldsymbol{\sigma}_\odot$ and $\hat{T}_T^{(\odot)} \equiv \tau_\odot^z [\hat{\mathbf{r}} \hat{\mathbf{r}} \cdot \boldsymbol{\sigma}_\odot - \frac{1}{3} \boldsymbol{\sigma}_\odot]$, $\tau_\odot = \tau_1 \odot \tau_2$, $\boldsymbol{\sigma}_\odot = \boldsymbol{\sigma}_1 \odot \boldsymbol{\sigma}_2$, where $\odot = \pm, \times$. Strictly speaking, $\boldsymbol{\mu}_{ij}^{1\pi}$ contains not only NLO but also admixtures of N³LO and higher order terms; if we expand $\boldsymbol{\mu}_{ij}^{1\pi}$ in powers of q , the q -independent part is NLO while the remaining q -dependent part is N³LO (or of even higher orders). Furthermore, if q is non-zero, as is the case for the np capture, Fourier transformation for the *soft*-one-pion-exchange current becomes rather involved, generating complicated N³LO contributions, which are denoted by the ellipsis in the above equation and will be reported elsewhere [24]. A numerical evaluation shows that the contribution of the N³LO part in the 1π current is negligibly small.

There are no corrections at N²LO, and all the contributions up to N³LO are included in this work. Among the N³LO contributions, the one-loop vertex correction of the one-pion exchange has been investigated in detail in Ref. [10, 11],

$$\begin{aligned}\boldsymbol{\mu}_{12}^{1\pi C} &= -\frac{g_A^2}{8f_\pi^2} (\bar{c}_\omega + \bar{c}_\Delta) \left[(\hat{T}_S^{(+)} + \hat{T}_S^{(-)}) \frac{\bar{y}_{0\Lambda}^\pi}{3} + (\hat{T}_T^{(+)} + \hat{T}_T^{(-)}) y_{2\Lambda}^\pi \right] \hat{j}_1(qR) \\ &\quad + \frac{g_A^2}{8f_\pi^2} \bar{c}_\Delta \left[\frac{1}{3} \hat{T}_S^{(\times)} \bar{y}_{0\Lambda}^\pi - \frac{1}{2} \hat{T}_T^{(\times)} y_{2\Lambda}^\pi \right] \hat{j}_1(qR) \\ &\quad - \frac{1}{16f_\pi^2} \bar{N}_{WZ} \tau_1 \cdot \tau_2 [\boldsymbol{\sigma}_+ \bar{y}_{0\Lambda}^\pi + (3\hat{\mathbf{r}} \hat{\mathbf{r}} \cdot \boldsymbol{\sigma}_+ - \boldsymbol{\sigma}_+) y_{2\Lambda}^\pi] \hat{j}_1(qR).\end{aligned}\quad (6)$$

The values of the LECs, $(\bar{c}_\omega, \bar{c}_\Delta, \bar{N}_{WZ})$, should in principle be fixed either by solving the underlying theory, QCD, or by fitting to suitable experimental observables. Since this has not yet been done, we adopt here the estimates given in Ref. [10, 11] based on the resonance saturation assumption and the Wess-Zumino action, $(\bar{c}_\omega, \bar{c}_\Delta, \bar{N}_{WZ}) \simeq (0.1021, 0.1667, 0.02395)$. Further discussion on the use of these estimates will be given later in the text.

The “fixed-term” contributions, $\boldsymbol{\mu}_{ij}^{1\pi:fixed}$, represent vertex corrections to the *soft*-one-pion-exchange and fixed completely by Lorentz covariance. They might be viewed as relativistic corrections to the πNN and $A_{\text{em}}^\mu \pi NN$ vertices. These corrections can be obtained conveniently by performing the Foldy-Wouthuysen transformation of the relativistic Lagrangian, and the resulting vertex functions read

• $\pi(k) + N(p) \rightarrow N(p')$ vertex :

$$\begin{aligned}&\frac{g_A}{2f_\pi} \tau^a \left\{ \boldsymbol{\sigma} \cdot \mathbf{k} - \frac{k^0}{2m} \boldsymbol{\sigma} \cdot (2\mathbf{p} + \mathbf{k}) \right. \\ &\quad \left. + \frac{1}{8m^2} [4\mathbf{p} \cdot \mathbf{k} \boldsymbol{\sigma} \cdot \mathbf{p} - 4\mathbf{p}^2 \boldsymbol{\sigma} \cdot \mathbf{k} - 2\mathbf{p} \cdot \mathbf{k} \boldsymbol{\sigma} \cdot \mathbf{k} + 2\mathbf{k}^2 \boldsymbol{\sigma} \cdot \mathbf{p} - \mathbf{k}^2 \boldsymbol{\sigma} \cdot \mathbf{k}] \right\}.\end{aligned}\quad (7)$$

• $\pi^a(k) + N(p) \rightarrow N(p') + A_\mu^{\text{em}}(q)$ vertex :

$$\begin{aligned}
& -i \frac{g_A}{2f_\pi} \epsilon^{3ag} \tau^g \left\{ \boldsymbol{\sigma} \left(1 - \frac{1}{8m^2} [4\mathbf{p}^2 - 4\mathbf{p} \cdot (\mathbf{q} - \mathbf{k}) + 2\mathbf{k}^2 - 2\mathbf{k} \cdot \mathbf{q} + \mathbf{q}^2] \right) \right. \\
& \quad \left. + \frac{1}{4m^2} \left[\mathbf{p}\boldsymbol{\sigma} \cdot \mathbf{p}' + \mathbf{p}'\boldsymbol{\sigma} \cdot \mathbf{p} + \frac{1}{2}\mathbf{k}\boldsymbol{\sigma} \cdot (2\mathbf{q} - \mathbf{k}) - \frac{1}{2}\mathbf{q}\boldsymbol{\sigma} \cdot \mathbf{k} + i\mathbf{k} \times \mathbf{p} \right] \right\} \\
& + \frac{g_A}{2f_\pi} (\tau^a + \delta^{3a}) \left\{ \frac{k_0}{2m} \boldsymbol{\sigma} + \frac{1}{4m^2} [\mathbf{k} \times (\bar{\mathbf{p}} \times \boldsymbol{\sigma}) + \boldsymbol{\sigma} \times (\bar{\mathbf{p}} \times \mathbf{k}) + i\mathbf{q} \times \mathbf{k}] \right\}. \quad (8)
\end{aligned}$$

The expressions of the M1 operator corresponding to the fixed terms are quite lengthy and will be reported elsewhere [24]. The fixed terms containing the nucleon momentum operators make the calculation highly involved.

The two-pion-exchange diagrams shown in Fig.2 give rise to

$$\begin{aligned}
\boldsymbol{\mu}_{12}^{2\pi} &= \frac{1}{128\pi^2 f_\pi^4} \left[\left(\hat{T}_S^{(+)} - \hat{T}_S^{(-)} \right) L_S + \left(\hat{T}_T^{(+)} - \hat{T}_T^{(-)} \right) L_T \right] \hat{j}_1(qR) \\
&- \frac{1}{256\pi^2 f_\pi^4} (\tau_1 \times \tau_2)^z \mathbf{R} \times \hat{\mathbf{r}} \frac{d}{dr} L_0 \hat{j}_0(qR), \quad (9)
\end{aligned}$$

where

$$\begin{aligned}
L_S &= -\frac{g_A^2}{3} r \frac{d}{dr} K_0 + \frac{g_A^4}{3} [4K_1 - 2K_0 + r \frac{d}{dr} (K_0 + 2K_1)], \\
L_T &= \frac{g_A^2}{2} r \frac{d}{dr} K_0 + \frac{g_A^4}{2} [4K_T - r \frac{d}{dr} (K_0 + 2K_1)], \\
L_0 &= 2K_2 + g_A^2 (8K_2 + 2K_1 + 2K_0) \\
&\quad - g_A^4 (16K_2 + 5K_1 + 5K_0) + g_A^4 \frac{d}{dr} (rK_1). \quad (10)
\end{aligned}$$

The loop functions K' s are defined in Ref. [8, 10].

Finally there are contact-term contributions of the form

$$\boldsymbol{\mu}_{12}^{CT} = \frac{1}{2m_p} [g_{4S} (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) + g_{4V} T_S^{(\times)}] \delta_\Lambda^{(3)}(\mathbf{r}) \hat{j}_0(qR), \quad (11)$$

where $g_{4S} = m_p g_4$ and $g_{4V} = -m_p (G_A^R + \frac{1}{4} E_T^{V,R})$; g_4 , G_A^R and $E_T^{V,R}$ are the coefficients of the contact terms introduced in Refs. [10, 11]. A noteworthy point is that, after removing redundant terms, there are only two independent contact-terms relevant for the M1 operator up to N³LO. This reduction of the effective number of counter terms is due to Fermi-Dirac statistics; a similar reduction has been noticed for the Gamow-Teller operator [8], where only one linear combination of LECs needs to be retained.

3 Results

We now use the M1 operators described above to calculate the magnetic moments of ³He, ³H and ²H as well as the rate of radiative capture of a thermal neutron

on a proton, $np \rightarrow d\gamma$. The total $np \rightarrow d\gamma$ cross section at threshold reads

$$\sigma_{np} = \frac{q^3}{v_n} \mu_N^2 M_{np}^2 \quad (12)$$

with

$$M_{np} = 2m_p \int d^3\mathbf{r} \psi_{d,0}^\dagger(r) \boldsymbol{\mu}^z(\mathbf{q}) \psi_{np}(r), \quad (13)$$

where $v_n = 2,200$ m/s denotes the neutron velocity in the lab frame, $\psi_{d,0}(r)$ the deuteron wave function with the spin component $J_z = 0$. The $\psi_{np}(r)$ denotes the spin-singlet np scattering wave function obtained by applying the normalization condition $\lim_{r \rightarrow \infty} u_{np}(r) = r - a_s$, where a_s is the spin-singlet np scattering length. The experimental value of the radiative np capture cross section is $\sigma_{np}^{exp} = 332.6 \pm 0.7$ mb [25], which corresponds to $M_{np}^{exp} = 410.2 \pm 0.4$ fm^{3/2}.

A realistic nuclear Hamiltonian is constructed using a high-precision phenomenological potential. We consider here the Av18 bi-nucleon potential [26] with/without the Urbana IX (U9) tri-nucleon force (3NF) [27], and we also consider the Argonne Av14 bi-nucleon potential with/without the Urbana VIII tri-nucleon force (U8). We remark that, from the HBChPT point of view, 3NF is N³LO compared to the leading nucleon-nucleon interactions, and hence should be included in our N³LO calculation.

The variational Monte Carlo (VMC) technique [27, 28] is used to solve the 3-body Schroedinger equation in order to obtain wave functions of ³H and ³He. MEEFT is based on the assumption that the adopted nuclear wave functions are exact to the order under consideration. This presupposes (i) the validity of using the high-precision phenomenological potential instead of an EFT-based potential, and (ii) the sufficient accuracy of the method used for solving the A-body Schroedinger equation for a given phenomenological A-body Hamiltonian. The first point is a fundamental issue that awaits further detailed studies. As for the second point, we remark that the long-range behavior of the wave functions can significantly affect the calculated nuclear transition matrix elements, and hence one should avoid using schematic wave functions. Unfortunately, it is difficult to fully quantify the uncertainty related to the wave functions used in this work. One of indirect measures for the accuracy of the adopted potentials and the VMC technique is the binding energies of ³H and ³He. We give in Table 1 the results obtained in VMC for the Av18 and Av18+U9 potentials; also shown in the table are the results obtained by solving the Faddeev equations [29, 30, 31]. Note that the use of 3NF is imperative in order to reproduce the binding energies of ³H and ³He. Later in the text we shall give comparison of the results for the M1 observables calculated with the Av18(+U9) potential and the Av14(+U8) potential, and this comparison provides another indirect measure for the stability of our results against changes in the input potential.

The cutoff Λ has a physical meaning, and its choice is not arbitrary [32]. Thus Λ should be smaller than the masses of the vector mesons that have been integrated out. Meanwhile, since the pion is an explicit degree of freedom in our scheme, Λ should be much larger than the pion mass in order to ensure that all pertinent low-energy

Table 1: The binding energies (in MeV) of ${}^3\text{H}$ and ${}^3\text{He}$ calculated in the VMC method for the Av18 and Av18+U9 potentials. For comparison, we also give in the square brackets the results obtained by solving the Faddeev equations.

	Av18	Av18+U9	Exp
BE(${}^3\text{H}$)	7.35(1) [7.61]	8.24(1) [8.47]	8.48
BE(${}^3\text{He}$)	6.59(1) [6.91]	7.48(1) [7.74]	7.72

Table 2: Magnetic moments of ${}^3\text{H}$ and ${}^3\text{He}$ in units of the nuclear magneton calculated for the Av18+U9 interactions. Also listed are the values of g_{4s} and g_{4v} which, for a given value of Λ , reproduce the experimental values of $\mu({}^2\text{H})$ and M_{np} . The parenthesized numbers represent the Monte Carlo statistical errors, while the entries in the square brackets correspond to the calculations in which the contact term contributions are ignored.

Λ [MeV]	$\mu({}^3\text{H})$	$\mu({}^3\text{He})$	g_{4s}	g_{4v}
500	3.034(13) [2.883]	-2.196(13)[-2.074]	0.5786	2.8790
600	3.035(13) [2.944]	-2.198(13)[-2.120]	0.2995	1.9567
700	3.036(13) [2.988]	-2.199(13)[-2.150]	-0.0202	1.2954
800	3.037(13) [3.019]	-2.200(13)[-2.168]	-0.3965	0.7882

Table 3: The magnetic moments of ${}^3\text{He}$, ${}^3\text{H}$, ${}^2\text{H}$ (in units of μ_N) and the $np \rightarrow d\gamma$ matrix elements (in units of $\text{fm}^{3/2}$) calculated with the Av18+U9 potential for $\Lambda = 600$ MeV.

	$\mu({}^2\text{H})$	M_{np}	$\mu({}^3\text{H})$	$\mu({}^3\text{He})$
LO: 1B	0.8469	393.1	2.585	-1.774
NLO: 1π	0.0000	8.7	0.205	-0.205
N ² LO: 1B $1/m^2$	-0.0069	-0.1	-0.018	-0.007
N ³ LO: $1\pi C$	0.0077	4.4	0.133	-0.116
N ³ LO: fixed term	0.0044	-0.2	-0.003	0.014
N ³ LO: 2π	0.0000	1.5	0.043	-0.043
N ³ LO: g_{4s} term	0.0053	0.0	0.007	0.007
N ³ LO: g_{4v} term	0.0000	2.9	0.085	-0.085
N ³ LO total	0.0174	8.6	0.265	-0.223
total	0.8574	410.2	3.035	-2.198
experiment	0.8574	410.2(4)	2.979	-2.128

contributions are properly included. In the present work, we consider $\Lambda = 500, 600, 700$ and 800 MeV as representative values.

For a given value of Λ , we adjust g_{4s} and g_{4v} to reproduce the experimental values of $\mu({}^2\text{H}) = 0.8574\mu_N$ and σ_{np}^{exp} (or equivalently M_{np}^{exp}). We remark that the fitted values of g_{4s} and g_{4v} depend on a particular choice of the potential model used to calculate the wave functions as well as the cutoff parameter. In Table 2, we show our predictions for the ${}^3\text{H}$ and ${}^3\text{He}$ magnetic moments obtained with the Av18+U9 potential for the four representative values of Λ ; also shown are the values of g_{4s} and g_{4v} optimized for each value of Λ . To highlight the roles of the contact terms, we include in the table the results corresponding to cases for which the contact terms are artificially dropped; see the entries in the square brackets. Table 2 indicates that the results of the full calculation are almost independent of Λ , whereas those obtained without the contact-term contributions show pronounced cutoff dependence.

Table 3 shows the contributions of the individual M1 operators to the magnetic moments of ${}^3\text{He}$, ${}^3\text{H}$ and ${}^2\text{H}$ as well as the matrix element for $np \rightarrow d\gamma$, calculated with the Av18+U9 potential for $\Lambda = 600$ MeV. The table indicates that the NLO 1π contribution is rather small, comparable to the total N³LO contribution. Substantial cancellation among the various terms contributing to $\mu^{1\pi}$ is responsible for this feature. Thus the $\mu^{1\pi}$ contribution to M_{np} , 8.7 in units of $\text{fm}^{3/2}$, can be decomposed as $8.7 = (19.4 - 14.4 + 4.0) - 0.3$, where the first three terms come from the first line of eq.(4), and the fourth term from the second line.

Finally, in Table 4 we compare the results for the different choices of the nuclear potentials; the table also gives the results of the SNPA calculations taken from Refs. [33, 34], as well as the experimental data [35]. The numbers in the parentheses represent statistical errors of our VMC calculation. We remark that the uncertainty in the experimental value of σ_{np} , which affects the determination of g_{4s} and g_{4v} , causes

Table 4: The $A = 3$ magnetic moments (in units of μ_N) calculated for $\Lambda = 600$ MeV. The last column gives the values of g_{4s} and g_{4v} fitted to reproduce the experimental values of $\mu(^2\text{H})$ and M_{np} . The bottom three rows show the SNPA results and the experimental data.

	$\mu(^3\text{H}) + \mu(^3\text{He})$	$\mu(^3\text{H}) - \mu(^3\text{He})$	g_{4s}	g_{4v}
This work (Av18+U9)	0.838(0)	5.233(25)	0.300(6)	1.96(29)
(Av18)	0.838(0)	5.242(26)	0.300(6)	1.96(29)
(Av14+U8)	0.838(0)	5.266(33)	0.391(7)	2.25(31)
(Av14)	0.844(0)	5.204(30)	0.391(7)	2.25(31)
SNPA I [33]	0.828	5.078		
SNPA II[34]	0.884	5.114		
Experiment[35]	0.851	5.107		

~ 0.5 % uncertainty in $\mu(^3\text{H}) - \mu(^3\text{He})$. Our results are found to be almost independent of the interaction model and the cutoff values used, satisfying the general tenet of EFTs that the model- and cutoff-dependence should be of higher order than the order to which calculation is performed, i.e., N^3LO for the present case. This potential-independence is also consistent with the notion of the V_{low-k} [36], which dictates that all the high-precision phenomenological potentials become universal if the cutoff is lowered to 2 fm^{-1} . Physically, this reflects the fact that the differences among various high-precision potentials lie only in the high-energy region that can be renormalized away leaving little effect for low-energy dynamics, provided that the renormalization procedure is correctly done.

Comparison of the results with and without the tri-nucleon forces (3NF) in Table 4 indicates that the role of 3NF in the magnetic moments is small; the largest effect seen for the Av14+U8 case is of the level of 1%.

Our no-parameter MEEFT calculation gives the values of $\mu(^3\text{H}) + \mu(^3\text{He})$ and $\mu(^3\text{H}) - \mu(^3\text{He})$ which agree with the experimental values at the 2-3 % level. This is slightly worse than the agreement reported in the SNPA calculations. We will come back to this point in the next section. We emphasize, however, that as far as the structure of the M1 operators is concerned, the present work gives a complete expression, a feature that distinguishes MEEFT from SNPA.

4 Discussion

We have reported here the calculation of the magnetic moments of ^3H and ^3He based on HBChPT. All the M1 operators up to N^3LO have been explicitly derived. At N^3LO , two unknown parameters, g_{4s} and g_{4v} , enter as the coefficients of contact terms. Following the MEEFT strategy, we have fixed them by imposing the renormalization condition that the experimental values of the deuteron magnetic moment and thermal neutron capture rate on proton be reproduced at N^3LO .

Concerning the $2 \sim 3$ % discrepancy between the values of $\mu(^3\text{H}) + \mu(^3\text{He})$ and $\mu(^3\text{H}) - \mu(^3\text{He})$ obtained in our MEEFT calculation and the corresponding experimental values, there are a few points to be discussed. The first is the level of accuracy of the $A = 3$ wave functions obtained in the VMC method. The indication that the VMC wave functions deviate slightly from the accurate wave functions is already visible in Table 1, which shows that the binding energies of the $A = 3$ systems calculated in VMC do not quite agree with those obtained in the Faddeev calculation [29, 30, 31]. An attempt is in progress [24] to improve the present work with the use of *ab initio* wave functions obtainable in the Faddeev method and to extend our formalism to the $^3\text{He} + n \rightarrow ^4\text{He} + \gamma$ reaction.

The second point is higher order contributions. As can be seen in Table 3, the difference between theory and experiment is only about 30 % of N^3LO contributions, for both isovector and isoscalar channel. Given the $\text{HB}\chi\text{PT}$ expansion parameter, this suggests that N^4LO effects of natural size can remedy the discrepancy. Among the N^4LO contributions, we expect the three-body (3B) currents – which appear first at N^4LO and hence have not been studied here – to play the most important role for the following reasons. We note that a substantial portion of the higher-order two-body contributions can be effectively absorbed into the renormalization of the coefficients of the contact terms; this is particularly true for short-ranged contributions. To illustrate this aspect, we consider the role of the 2π contribution in our N^3LO calculation and compare the results of two “ 2π -contribution-less” calculations. In the first case, the 2π terms are simply dropped without any other accompanying changes, whereas in the second case the values of g_{4s} and g_{4v} are readjusted to reproduce the experimental values of $\mu(^2\text{H})$ and σ_{np} (*without* the 2π contributions). We expect that the change in the net results for the second case, which may be viewed as an *effective* 2π contribution, is much smaller than the 2π contribution estimated from the first case. We have verified this feature numerically. From this example, we may expect that the *effective* N^4LO 2B contributions are small. The situation is however completely different for the 3B contribution in the M1 operator, ($\mu_{3\text{B}}$). Since $\mu_{3\text{B}}$ appears in the $A = 3$ systems but not in the $A = 2$ systems, its contribution cannot be absorbed into the two-nucleon contact terms. The higher order calculation up to N^4LO is relegated to future work.

Finally, we discuss the uncertainty related to the values of the LECs, \bar{c}_ω , \bar{c}_Δ and \bar{N}_{WZ} , which appear in $\mu_{12}^{1\pi C}$, eq. (6). The $\mu_{12}^{1\pi C}$ contributions themselves are comparable in size to the difference between the present calculation and the experimental data, as can be seen in Table 3. However, due to the above-mentioned renormalization procedure, the *effective* $\mu_{12}^{1\pi C}$ contributions are expected to be small. We have checked this by turning off $\mu_{12}^{1\pi C}$ and re-performing the renormalization procedure, and found that the *effective* $\mu_{12}^{1\pi C}$ contributions are negligible. If on the other hand we naively treat these LECs as free parameters and adjust them to reproduce the experimental values, we are led to very large values of the LECs, which strongly violates the naturalness condition.

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